Dice Cubes: Quantum-Based Networked Performance with CubeHarmonic

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***Abstract*—This paper explores the intersection of quantum computing and the Internet of Musical Things, a topic that hitherto has received little attention from scholars. The CubeHarmonic (CH) is a musical instrument based on the Rubik's cube. Here, we consider the case of three remote performers of CH that exchange over the network minimal information, as a string of probability amplitudes of performing specific rotations. The state of possible rotations are modeled as quantum states, and the actual decided moves as results of a quantum measurement. We propose a quantum circuit to help maximize the musical variety that can be obtained by three remote CH performers.**

***Index Terms*—Internet of Musical Things, Networked Music Performance, Quantum Computing**

I. Introduction

The Rubik's cube [1] has recently celebrated the fiftieth anniversary [2]. Among its many applications, as mentioned in [2], there is a musical instrument, the *CubeHarmonic* (see Fig. 1). Such instrument was conceived by Maria Mannone in 2013 [3], and was developed at the laboratory of Yoshifumi Kitamura in Japan [4]. Its 4D version has been developed in collaboration with Takashi Yoshino [5].

The CubeHarmonic, in short CH, is a Rubik's cube having pitches on its faces. Scrambling the cube, one gets different musical chords. Thus, mathematical relationships between musical intervals, as formalized on the plane with the tonnetz [6], can be joined with the mathematics of combinatorics and group theory. CH was inspired by the Musical Dice Game of Mozart (Musikalisches Würfelspiel), Llull, Haydn and the slot-machine transformations, that is, sets of permutations applied to single notes in musical chords, as the permutations in slot machines [7], [8]. The notation used to indicate the rotations on the Rubik's cube [1] can be adapted to the CH, as a tablature system. In current development, CH is a completely virtual or half-virtual instrument [5].

In the context of the Internet of Musical Things [9], CH can be used in conjunction with a networked music performance (NMP) system [10]–[12], leading to performances over the network where CH players play together at a distance. Within this context, it is possible to explore novel interaction possibilities among connected CH performers, which are mediated by quantum computing.

Quantum computing is an emerging technology founded on the principles of subatomic physics [13]. This technology has recently made its inroads in musical applications and such synergy is gaining momentum. This is evidenced by the emergence of the Quantum Computer Music research field [14], an increasing number of publications on the subject [15]–[18] and dedicated international gatherings[1]. However, to date, the intersection of quantum computing and the Internet of Musical Things has received little attention from the research community. A noticeable exception is represented by our previous study reported in [19], where a decision-making system centered around a quantum circuit was proposed for the case of NMP: performers' approximated decisions were modeled using state superposition and probability amplitudes from quantum computing. Specifically, the model accounted for factors such as signal clarity (audio quality affected by packet losses), latency, and musical novelty (melodic or harmonic variation relative to a previous musical sequence) that influence a performer's decision to follow another connected performer.

Here, we explore the integration of CH, NMP systems and quantum computing. Joining *cubes* and *qubits*, the potential moves of CH are expressed as quantum states with probability amplitudes, measured during the decision step, result transmitted between remote CH performers.

The motivation of our study is aesthetic and conceptual. We explore the potentiality of a multi-CH performance. We also add one more layer of complexity, with the use of quantum computing, maintaining a minimal amount of information exchanged: only a string of three numbers. In addition, the intrinsic-probability of quantum mechanics is used here to model performers' thinking (inspired by [20], [21]), as a dialogue with the actual choice of moves. The article provides a theoretical framework and an example of application. We aim to use the Rubik's cube as an instrument to generate harmonic

[1] https://iccmr-quantum.github.io/1st_isqcmc/

complexity between performers, not to be solved as a puzzle. We propose a harmonic kaleidoscope and a mathematical exploration.

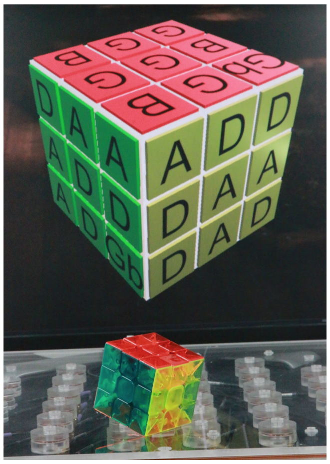

Fig. 1. CubeHarmonic, in its version with 3D-printed cube, embedded motion sensor, magnetic tracking system IM3D [22], and pitch rotations visualized in virtual reality [3]. Credits: Y. Kitamura, 2017.

II. Experiment

A. Quantum notation

Let us consider three CH performers. Each of them can rotate a slice of the cube, along three directions: left/right, top/down, and front/back. According to the literature on the Rubik's cube, these rotations are identified as, respectively: L/R, T/D, F/B [1]. We can define a space of rotations as a space x, y, z. If we associate a state 0 along axis x with L, state 1 along x with R, and so on, and the probability amplitudes to perform a rotation with Greek letters, borrowing the formalism from quantum mechanics, for the i-th player we can define the following states:

$$\begin{aligned} |\psi\rangle_{i,x} &= \alpha_i|L\rangle + \alpha_i'|R\rangle \\ |\psi\rangle_{i,y} &= \beta_i|T\rangle + \beta_i'|D\rangle \\ |\psi\rangle_{i,z} &= \gamma_i|F\rangle + \gamma_i'|B\rangle, \end{aligned} \tag{1}$$

where α_i is the probability amplitude to turn R, α_i' the probability amplitude to turn L, with $|\alpha_i|^2 + |\alpha_i'|^2 = 1$, similarly $|\beta_i|^2+|\beta_i'|^2 = 1$, $|\gamma|^2+|\gamma_i'|^2 = 1$. The probability amplitude β_i refers to a turn [check grammar] of the upper face clockwise, and γ_i to a turn the front face clockwise. Changes of sign indicate the corresponding rotation but counterclockwise.

B. Toy example

Let us consider three CH tuned with the same note on each face, that is, the same note on each color. Let we have E (top), C (right), A (front), B (left), F (back), and D (down), see Figure 2.

Let the cube play the sound(s) corresponding to the front face. At the beginning, there is only the A sound. If the first performer turns the right slice clockwise, we hear A and D.

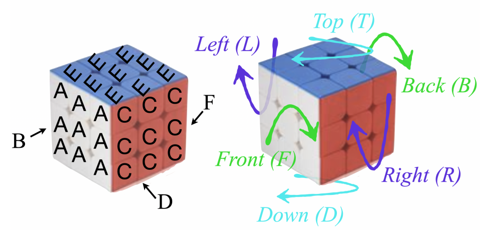

Fig. 2. A simple tuning for the cube (left), and illustration of the possible rotations (right).

The signal that the first performer sends to the other two performers is thus:

$$(\text{Right}, \mathbb{I}, \mathbb{I}) \Rightarrow \{\alpha_1, \beta_1, \gamma_1\} = \{1, 0.5, 0.5\}\,, \tag{2}$$

where the identity indicates that no rotation has been performed, and where we adopt the following notation rule:

- if the probability amplitude > 0.5, rotate clockwise as state 0;
- if < 0.5 and > 0, rotate clockwise as 1;
- if $= 0.5$, do nothing;
- if the module of probability amplitude > 0.5 but < 0, rotate counterclockwise as state 0;
- if the module < 0.5 but < 0, rotate counterclockwise as 1.

Let us imagine that the other two performers do the following, respectively:

- 2: $\{0, 0.5, 0.5\}$
- 3: $\{0.5, 1, 0.5\}$,

then they get from A to A, E, and from A to A, C, respectively. If each cube plays the notes on the front face, then the harmony is enriched as follows:

$$\begin{aligned} 1 &: A \to A, D, \\ 2 &: A \to A, E, \\ 3 &: A \to A, C, \end{aligned} \tag{3}$$

thus, from a single note, a chord A, C, E, D is obtained. Considering the rotations performed one after the other, emphasizing in loudness the most recent rotations, one can also hear an emerging melody as shown in Figure 3.

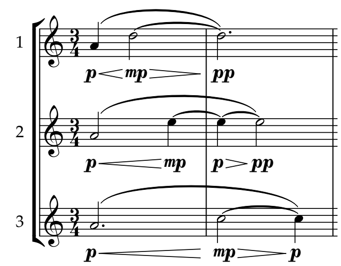

Fig. 3. A simple polyphony from the sequence of rotations of Eq. (3), with the emerging melody A - D - E - C. Each pentagram indicates a CubeHarmonic. The rhythm here is even; however, fluctuations due to latency are expected.

Enriching each cube of different pitches on each face, timbres for each colors, or even including rhythmic patterns in some of the facets, one can obtain musical sequences of increasing complexity, having a control of the initial material and of the rotations performed.

C. Thinking, decision, and communication

The mental state of each performer can be modeled with a quantum state composed by three qubits, each of them associated with the one of the rotations described above.

Each rotation of the cube can be seen as a transformation along one of the coordinates described above.

The decision of each of the performers can be modeled as the result of a quantum measurement, that lets the state collapse to a specific combinations of probability amplitudes. The information sent by the i-th performer to the other two performers, is thus a string $\alpha_i, \beta_i, \gamma_i$, where they are the results of the measurement. Conceptually, the quantum formalism can help model mental states [20], representing the transition from thinking to action as an operation of measure [21]. Thus, a collaborative artistic performance can be seen as an interplay between quantum and classic world [21].

D. Results

To increment the variety of chordal combinations, we can sketch a quantum circuit, that takes as input the string sent by a performer, and gives as output a suggestion for the other two performers. Or, that takes as input the string of two performers, and gives as output the suggestion of rotations to the third performer. We focus here on the second case: the moves of first and second player, through a quantum circuit, suggest a choice of moves for the third player.

We can schematize the idea with a logic gate, whose inputs are the strings of the first two performers, and whose output is the suggestion of moves, as a string, for the third performer. At each step of the playing, the role of the three performers can be exchanged, e.g., with second and third giving suggestions to the first performer, and so on. Thus, we can see that the permutational idea recurs at different levels of the networked performance.

As a simple proof of concept, let us run the "quantum dice" for our cubes, using the Qiskit language and the Quantum Composer application provided by IBM.[2] We can remotely call a quantum computer and run simulations. Given the limit of 7 qubits for the free version of IBM Quantum Composer, we restrict our simulation to two axes for the three performers. Thus, we consider $|\psi\rangle_{1,x}$, $|\psi\rangle_{1,y}$, $|\psi\rangle_{2,x}$, $|\psi\rangle_{2,y}$, and $|\psi\rangle_{3,x}$, $|\psi\rangle_{3,y}$, that is, the three performers can rotate the left, right, top and down faces of the cube.

We consider again three CH tuned as described above. We define two qubits for each performer. All qubits are initialized with a Hadamard gate (H), to express the equal superposition of states up and down, that is, a probability amplitude of 0.5. Concerning the cube rotation, we consider this as an "undecided" state: a rotation that has not been performed (yet), that is, an unscrambled cube. For the first performer, we have:

[2] So far, CH does not input to Qiskit, but a direct connection can be developed.

- q_0: it represents $|\psi\rangle_{1,x}$; state down ($|0\rangle$, $\alpha_1 = 1$) indicates that the performer 1 made a rotation left (L); state down ($|1\rangle$, $\alpha_1 = 0$), rotation right (R);
- q_1: $|\psi\rangle_{1,y}$; the performer 1 makes a rotation of the top (T) layer ($|0\rangle$, $\beta_1 = 1$), or the bottom (B) layer ($|1\rangle$, $\beta_1 = 0$), rotation right (R).

Qubits q_2, q_3 are built similarly for the second performer, and qubits q_4, q_5 for the third performer. We thus build the

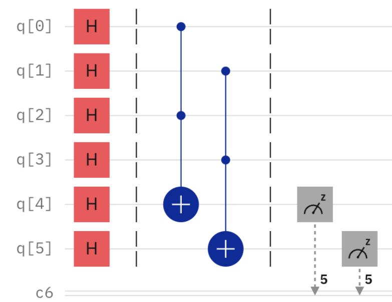

Fig. 4. Quantum circuit representing a quantum-informed 3-player performance with CubeHarmonics. The qubits represents the state of possible rotations, of left/right and top/bottom slices of the each cube. Qubits q_0, q_1 refers to performer 1, q_2, q_3 to performer 2, and q_4, q_5 to performer 3. $c6$ is the classic cube where the result of the measurement is stored. If the three cubes are initialized as unscrambled, all of them present H gates at the beginning.

quantum circuit shown in Figure 4. We use the Hadamard gate, as explained, to initialize the cubes in the unscrambled position. The circuit itself is contained inside the dashed lines (called barriers), and, in our most simple choice, is constituted by two Toffoli gates, the CCN (NOT with two controlling gates). It means that the target qubit's states is switched only if both the controlling qubits are 1. The two symbols after the second barrier indicate the measurement, whose result is stored in the classic qubit c. The circuit is run 1024 times, obtaining an histogram of the most frequent outcomes. The most frequent state is considered as the "suggested" move combination for the third performer. Adding more gates to the first and second cubes (qubits: 0 to 3), we represent some rotations performed on them; consequently, the suggested move for the third cube will change. Table I shows the results of a simulation with a quantum computer, called remotely, located in Osaka, accessed through the quantum services of IBM. We present three tests, the output of quantum simulation giving the most likely obtained combination, and the musical corresponding results. In the third test, the two states $[0, 1]$ and $[0, 0]$ present equal number of occurrences, thus we indicated $[0, 0.5]$ as the most likely output. When the quantum computer is making recommendations, this application could lead to a fully autonomous instrument. The results of our simple experiment show that we actually keep getting musical variety. That is still quite limited due to the limitation of the pitch material; however, besides the specific results, this is a way to show how to relate these inputs from different fields.

A pictorial representation of the proposed network is presented in Figure 5. There is a network constituted by three

TABLE I
THREE TESTS WITH THE CIRCUIT OF FIGURE 4. INSIDE THE CURLY BRACKETS, THERE ARE THE COEFFICIENTS; THE CORRESPONDING NOTES HEARD ARE ARE ALSO SHOWN BY SIDE.

input						output		
player 1			player 2			player 3		
string	moves	notes	string	moves	notes	most likely output	moves	notes
$\{1, 0.5\}$	(L, $\mathbb{I}$)	A→A, D	$\{0, 1\}$	(R, T)	A→A, C	$[0, 0] \to \{1, 1\}$	(L, T)	A→A, B, F
$\{0, 0.5\}$	(R, $\mathbb{I}$)	A→A,C	$\{1, 0\}$	(L, D)	A→A, B, D	$[0, 0] \to \{1, 1\}$	(L, T)	A→A, B, F
$\{0.5, 1\}$	($\mathbb{I}$, T)	A→A, C	$\{0.5, 0.5\}$	($\mathbb{I}$, $\mathbb{I}$)	A→A	$[0, 0.5] \to \{1, 0.5\}$	(L, $\mathbb{I}$)	A→A, B

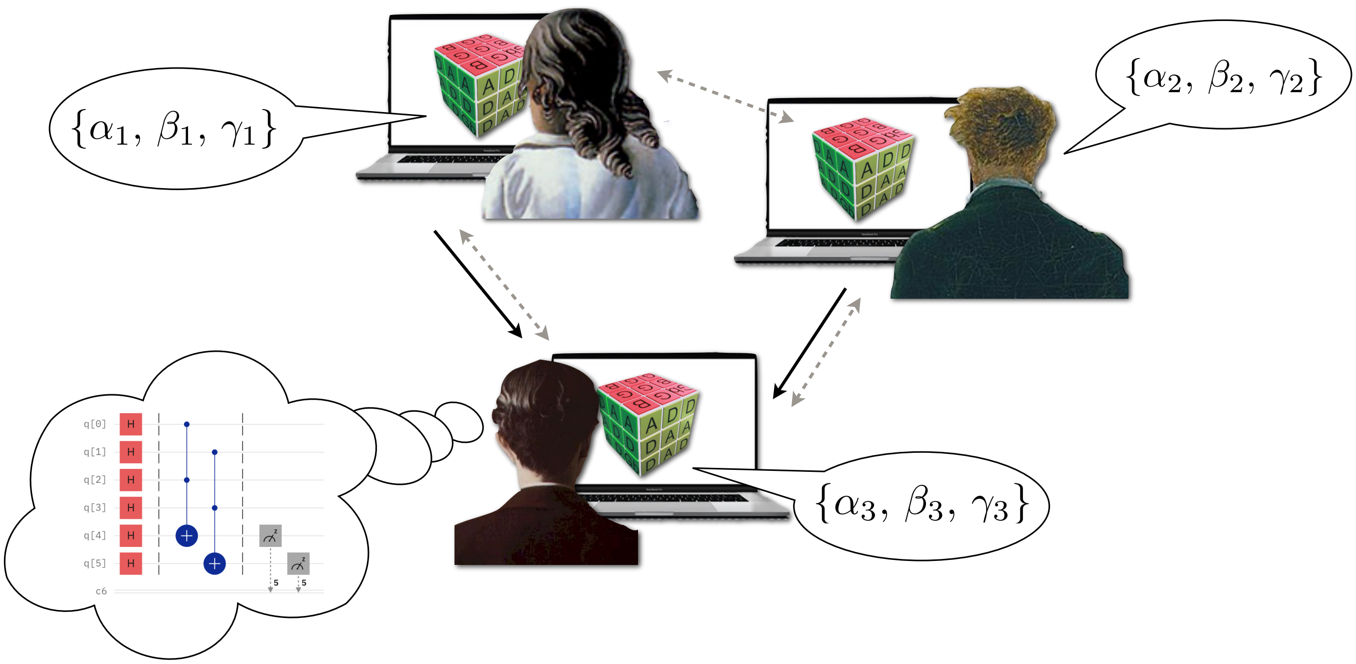

Fig. 5. A pictorial representation of the proposed network, with three remote human performers of CubeHarmonic. The black arrows indicate the signal transmission as in the considered example, from the first two performers to the third one. The gray dashed arrows indicate the other possible directions of signal transmission. $\{\alpha_i, \beta_i, \gamma_i\}$, $i = 1, ..., 3$, are the strings of information corresponding to the choice of rotations on the CubeHarmonic. After having received the strings from the other performers, the third one chooses his moves according to the most likely measurement from a quantum circuit, schematizing a thinking process to increase harmonic variety. Images of the CubeHarmonic: credits to Yoshifumi Kitamura. Images of the performers: credits to the painters Salvador Dalí (detail from *Young woman at a window*), Caspar David Friedrich (detail from *Wanderer above the sea of fog*), and René Magritte (detail from *Not to be reproduced*). Images of the laptops: Shutterstock. Graphic composition: Maria Mannone.

remote human performers, playing a virtual version of the CubeHarmonic, and exchanging information. In this scenario, the first two performers send their strings to the third performer, who decides which rotations to perform according to the more likely output of the quantum circuit of Figure 4. If, in a second step of the performance, the second and the third performers send their strings to the first one, she will choose a rotation or a set of rotations according to the output of the quantum circuit, and so on.

III. Discussion and Conclusions

The moves performed on three simulated distant CubeHarmonics, with simple movements from which a more and more harmonically complex music is emerging, as can be seen as the choices of individual robots in a swarm, whose decision-making system can be modeled via a quantum circuit [23]. Thus, the investigation of quantum-based networked performance with the CubeHarmonic can pave the way toward modeling in other areas of engineering, using sound as a probe to investigate complexity at large. Here, we have been throwing the dice multiple times across disciplines, involving combinatorics with the Rubik's cube, harmony + combinatorics with the CH, additional combinatorics with multiple CH, and additional variety with the definition of quantum states and their measurement for a simple decision-making system. Our setup requires a minimal information to be transmitted across performers, only a string of probability amplitudes. The advantage is a maximal musical variety with minimal exchanged information. The CH, and a multiple-CH networked remote performance as proposed here, is accessible also to visually-impaired cube solvers, thanks to the focus on pitches. There is no need of being able to solve the cube to enjoy the variety of produced music. In [4], the initial state (unscrambled) of CH could be restored thanks to the electronic association between physical facets (small squares) of the Rubik's cube and the pitches; in [5], the overall instrument is completely virtual, so the reset can be performed immediately as well. We also wonder if the logic puzzle be solved while achieving musicality. Musical creativity also involves the creation and the recombination of simple motivic, rhythmic, and harmonic material, and even dice have played a role. Distant performers do also recombine information and patterns. Quantum computing involves dice in a different way, and the Rubik's cube is based on permutation. We offered here a simple, yet dense, approach toward these fields, enhancing understanding, modeling, and ultimately, aesthetic enjoyment of science.

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